Summary sheet: Probability

M1 Understand and use mutually exclusive and independent events when calculating probabilities
Link to discrete and continuous distributions
N1 Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), as a model

Calculating probabilities

In probability, the word **experiment**, or **trial**, is any situation involving uncertainty.
An **outcome** is the result of a trial or experiment.
The set of all possible outcomes of a trial or experiment is called the **sample space**.
The word **event** can be used to describe one or more possible outcomes from a trial or experiment.

If all the possible outcomes are equally likely, then the probability of event A can be calculated.
So, for example, an experiment could be tossing a coin twice, and the outcome is the result of the coin tosses.
The sample space for this experiment is HH, HT, TH, TT.
If event A is 'getting one head and one tail' then the probability of A is given by $\mathrm{P}(A) = \frac{2}{4} = \frac{1}{2}$.

Sometimes a table is useful for finding a sample space. For example, if you spin two four-sided spinners, each numbered 1 – 4, you could use a table like this to find the totals of the two spinners:

		Spinner 1			
		1	2	3	4
Spinner 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

From the table, you can see that there are 16 possible outcomes. The total 5 occurs 4 times,
So $\mathrm{P}(\text{total} = 5) = \frac{4}{16} = \frac{1}{4}$.

Expected frequency

If you spin the two spinners (above) 100 times, the **expected frequency** of getting a total of 5 is $100 \times \frac{1}{4} = 25$. This does not mean that you would always get 25 instances where the total is 5, out of 100 spins!

Notation

- $\mathrm{P}(A)$ means 'the probability that event A occurs'.
- A' means 'the **complement** of event A', i.e. event A does not occur.
  So $\mathrm{P}(A') = 1 - \mathrm{P}(A)$.
- $\mathrm{P}(A \cup B)$ means 'the probability that either event A occurs, or event B occurs (or both)'.
- $\mathrm{P}(A \cap B)$ means 'the probability that both event A and event B occur'.

Summary sheet: Probability

Mutually exclusive events

Two events are **mutually exclusive** if it is impossible for them to occur together.
For example, if you throw two ordinary dice, and A is the event that the total of the dice is 8, and B is the event that one of the dice shows a 1, then A and B are mutually exclusive events, since if B occurs then to get a total of 8, the other dice would have to show a 7 which is not possible.

If two events are mutually exclusive, then

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$$

Independent events

Two events A and B are **independent** if whether or not A occurs has no effect on whether or not B occurs.

For example, if you toss a coin twice, then the outcomes of the two coin tosses are independent. However, if you have a bag containing 6 red balls and 4 black balls, and you take a ball out and then take another one out, the outcome of the second experiment depends on the result of the first, as the probabilities are different according to the outcome of the first experiment.

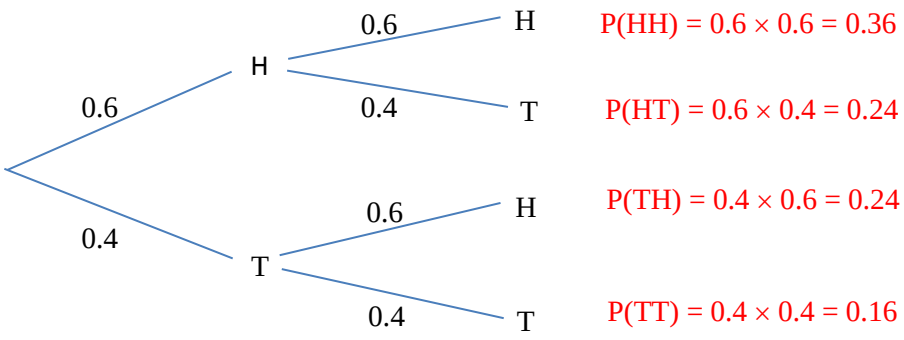
If events A and B are independent, then

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Using diagrams

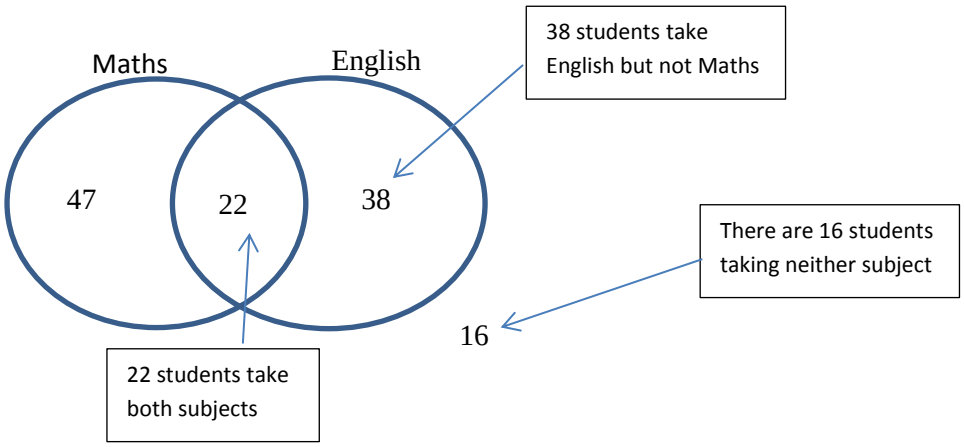
When you are working with more than one event, a diagram may help to work out probabilities.

- **Tree diagram**
  This is useful when you are dealing with events that have just two or three possible outcomes, such as tossing a coin. The tree diagram below shows the outcomes when a biased coin (probability of getting a head is 0.6) is tossed twice.



Summary sheet: Probability

- **Venn diagram**
  This is useful when you are dealing with events that are not mutually exclusive. The Venn diagram below shows the number of students taking Maths and / or English in a sixth form college.



Probability distributions

A random variable has a number (which may be infinite) of possible values, each of which has an associated probability. All the probabilities must add up to 1.
The probability distribution of a random variable gives the probability of each of the possible outcomes.

If X is the result of spinning two four-sided spinners, each numbered 1 – 4, the probability distribution of X could be written in a table like this:

x	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

