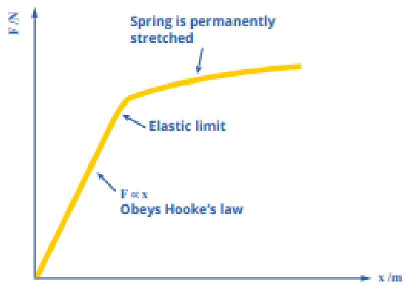
Module 3 Section 4: Materials

Hooke's Law

Hooke's law states that the tension in a spring or wire is proportional to its extension from its natural length, provided the extension is not too great.



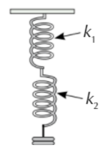
The relationship $F = kx$ can be used to calculate the extension up to the elastic limit. k is the spring constant and represents the stiffness of the spring.

For a wire with cross sectional area A, original length l and under tension F, you can determine **the stress, strain and Young modulus** using these equations. This information will allow you to compare the stiffness of different materials.

Hooke's Law and springs

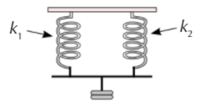
In series, the reciprocal of the combined force constant is equal to the sum of the reciprocals of the individual force constants:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



In parallel, the combined force constant is just the sum of the force constants of the individual springs:

$$k = k_1 + k_2$$



Force—extension graphs

Elastic potential energy is the energy of an object due to it being deformed by a force.

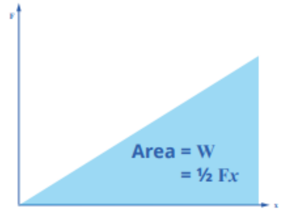
It can be calculated in two ways, either using this equation:

$$E = \frac{1}{2}kx^2$$

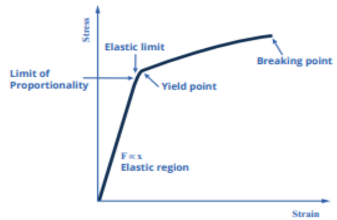
Using Hooke's law (see unit 1.5), this equation can be converted to,

$$E = \frac{1}{2}Fx = \frac{F^2}{2k}$$

Or calculated graphically using the area under a F-x graph.



Stress—strain: Ductile materials



In metals it is important to understand **edge dislocations** and **ductile fractures**. Edge dislocations are where there is an extra plane in the crystals, plastic deformation occurs when the **dislocations move** due to the large stress. Ductile fractures **(necking)** is where the number of edge dislocations increases and causes the elongation of the metal which increases the stress at the neck (smaller A).

Stress and Strain

Tensile stress, σ, is defined as the force applied, ***F***, divided by the cross-sectional area, *A*:

$$\sigma = \frac{F}{A}$$

The units of stress are Nm^{-2} or pascals, Pa.

A stress causes a strain. **Tensile strain**, ε, is defined as the change in length, i.e. the extension, divided by the original length of the material:

$$\varepsilon = \frac{x}{L}$$

The Young Modulus

$$\text{Young modulus} = \frac{\text{tensile stress}}{\text{tensile strain}} \qquad E = \frac{\sigma}{\varepsilon} = \frac{F \div A}{x \div L} = \frac{FL}{Ax}$$

Where ***F*** = force in N, *A* = cross-sectional area in m^2, *L* = initial length in m and ***x*** = extension in m. The units of the Young modulus are the same as stress (Nm^{-2} or Pa), since strain has no units.

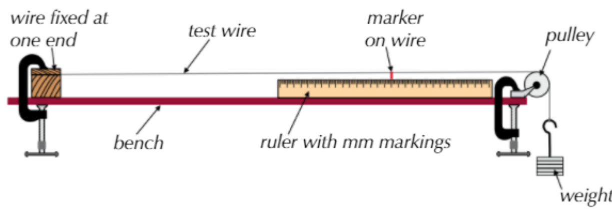


Figure 1: *Experimental set-up for determining the Young modulus of a test metal.*

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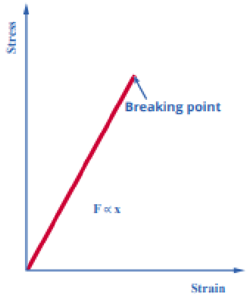
Stress-strain: Brittle materials

F-x graphs for brittle materials:

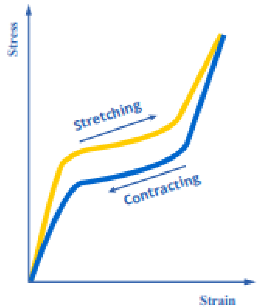
Brittle materials obey Hooke's law, although they do not stretch very much. These materials tend to fracture at a lower stress than expected, due to a process known as **brittle fracture**. This occurs when cracks in the surface of the materials magnify the stress at that point and cause the material to break. To avoid this, materials like concrete and pre-stressed glass are **always under compression** to stop cracks opening.



Stress-strain: Polymeric materials

F-x graphs for rubber:

Most important to note from this graph is that the stretching and contracting curves are different. This is **elastic hysteresis**. As the area under the curve = the work done, **more work is done when stretching than when contracting**. This means the extra energy used to stretch is transferred to vibrational energy in the rubber molecules.



Material Properties

Figure 9 shows the stress-strain curves for materials of different strengths and stiffnesses.

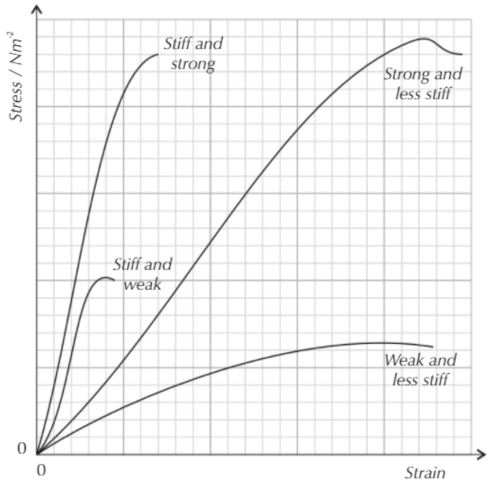


Figure 9: Stress-strain curves for different materials.

Measuring extension

Figure 5 shows the experimental set-up you could use in the lab to investigate how the extension of an object varies with the force applied to it.

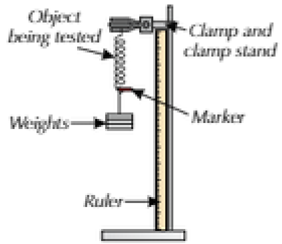


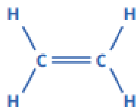
Figure 5: An experimental set-up used to investigate how extension varies with force.

Material Properties

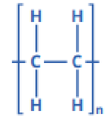
Amorphous materials e.g. glass and ceramics, have no long-range order.

Polymeric materials e.g. polythene and rubber, have very long molecules consisting of a large number of repeated monomers making one polymer.

Ethene monomer



Polythene repeat unit



Crystalline materials are solids with long-range order; the particles are arranged in a lattice.