Y12 Pure Chapter 1— Algebraic Expressions

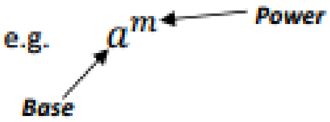
Indices (powers)

An index (power) tells you how many times to multiply something by itself:

e.g. x^5 means $x \times x \times x \times x \times x$

There is a base and a power:

e.g.



There are a few rules of indices that you need to learn and remember how to use:

Rule	Meaning	Example
$a^m \times a^n = a^{m+n}$	To **multiply** 2 numbers with the <u>same base</u> you **add** the powers.	$5^3 \times 5^4 = 5^7$
$\frac{a^m}{a^n} = a^{m-n}$	To **divide** 2 numbers with the <u>same base</u> you **subtract** the powers.	$\frac{3^7}{3^2} = 3^5$
$(a^m)^n = a^{m \times n}$	To simplify a power inside and outside of a **bracket** you **multiply** the powers.	$(6^4)^3 = 6^{12}$
$a^{-m} = \frac{1}{a^m}$	A negative power means **"one over"** so send everything to the bottom of a fraction.	$8^{-5} = \frac{1}{8^5}$
$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	A fractional power means a **root**. The bottom of the fraction tells you the root and the top tells you the power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$
$a^0 = 1$	**Anything** to the power of zero = 1	$57^0 = 1$

Finally

Remember that any number to the power of one stays the same: e.g. $72^1 = 72$

And 1 to the power of anything is 1: e.g. $1^{15} = 1$

$1^{-356} = 1$

Surds (roots)

A surd (root) is the inverse of a power:

e.g. $\sqrt{25}$ means "which number multiplied by itself would give 25?" the answer is 5 because 5x5=25.

Remember that a surd can be part rational, e.g. $(3+\sqrt{7})$ has a rational part (3) and the root part $(\sqrt{7})$.

It is a good idea to remember the first few perfect square numbers so that you can spot them when you are working with surds, (**1** (1x1), **4** (2x2), **9** (3x3), **16** (4x4), **25** (5x5), **36** (6x6) etc).

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified.

e.g. $\sqrt{20}$ can be re-written as $\sqrt{4}\times\sqrt{5}$ which simplifies to $2\sqrt{5}$

Perfect square

Adding and subtracting surds

Remember to add or subtract **like terms** (i.e. the rational numbers and the roots (of the same number))

e.g. $(7+3\sqrt{2})+(8-\sqrt{2}) = 15+2\sqrt{2}$

Add rational parts: ($7+8=15$)

Add roots: ($3\sqrt{2}-1\sqrt{2}=2\sqrt{2}$)

Multiplying surds

If there is no rational part then multiplying is easy: e.g. $\sqrt{3}\times\sqrt{5} = \sqrt{15}$

If there is a rational part then multiply out the brackets (either using FOIL (first, outside, inside, last) or the smile – whichever you prefer):

e.g. $(5+\sqrt{3})\times(2-\sqrt{3}) = 10-5\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ tidies up to give $7-3\sqrt{3}$

Remember that $\sqrt{3}\times\sqrt{3}=3$

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction. To rationalise the denominator just multiply the top and bottom of the fraction by the **bottom of the fraction with the opposite sign in front of the root.**

e.g. Simplify $\frac{3+\sqrt{5}}{2-\sqrt{5}}$ by rationalising the denominator.

Remember the rule: multiply the top and bottom of the fraction by the **bottom of the fraction with the opposite sign in front of the root.**

$$\frac{3+\sqrt{5}}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}} = \frac{11+5\sqrt{5}}{-1} = -11-5\sqrt{5}$$

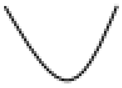
Notice these are the same – but the sign in front of the root has changed.

Changing the sign in front of the root makes the middle parts cancel each other out & disappear.

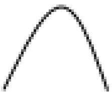
Y12 Pure Chapter 2— Quadratics

The general shape of quadratic graphs

$y = x^2$



$y = -x^2$



Solving quadratic equations

The two most common ways to solve a quadratic equation are **factorising** and using the **formula**. They will both give you the same answer so just use the most appropriate method.

Remember that to solve a quadratic equation you should collect all the terms on one side so that the other side of the equation is 0. When you solve the equation, it you have found the **roots** (i.e. where the graph of the quadratic function **crosses the x-axis**).

Solving by factorising

This means factorising the quadratic into 2 brackets.

e.g. Solve $x^2 + 6x + 8 = 0$ using factorisation.

$$x^2 + 6x + 8 = 0$$
$$(x + 4)(x + 2) = 0$$

Remember you need 2 numbers that multiply to give 8 and add to give 6.

Therefore: $x = -4$ or $x = -2$

Solving by using the formula

The quadratic formula:

When $y = ax^2 + bx + c$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e.g. Solve $x^2 + 6x + 7 = 0$ using the formula.

First of all, make a note that: $a = 1$
$b = 6$
$c = 7$

then substitute into the formula (put each number in a bracket so that you are careful to get the correct sign):

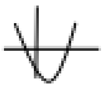
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{8}}{2} = -3 \pm \sqrt{2}$$

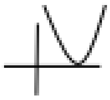
The Discriminant:

The expression inside the square root sign is called the discriminant and tells you what type of roots to expect.

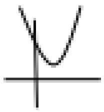
If $b^2 - 4ac > 0$ there are **2 real roots**
(i.e. the curve crosses the x-axis in 2 places)



If $b^2 - 4ac = 0$ there is **1 real root**
(i.e. the curve touches the x-axis in 1 place)



If $b^2 - 4ac < 0$ there are **no real roots**
(i.e. the curve does not cross the x-axis)



Completing the square to find the turning point

Remember that the completed square version of a quadratic equation looks like this:

$$a(x+b)^2+c=0$$

The turning point (maximum or minimum) will be at $(-b, c)$

e.g. Find the turning point of the equation $(x+5)^2-7=0$

The turning point (minimum) will be at $(-5, -7)$

e.g. Find the turning point of the equation $5-3(x-4)^2=0$

The turning point (maximum) will be at $(4, 5)$
Notice that the turning point is a maximum because the x^2 term is negative.

Don't forget:
If you need to solve the quadratic to find the roots and it is already in the completed square form, you don't need to factorise or use the formula you can just rearrange to find x.

Solving equations in a function of the unknown

Sometimes you may need to solve an equation which is a 'disguised quadratic'. This is an equation whch involves one term in which the power of x is twice the power of x in another term.

e.g. Solve the equation $x^4+3x^2+2=0$

The equation could be rewritten as $(x^2)^2+3x^2+2=0$
So you can let $y=x^2$ and you have: $y^2+3y+2=0$
Now you can solve by factorising.

e.g. Solve the equation $x^6+5x^3+6=0$

The equation could be rewritten as $(x^3)^2+5x^3+2=0$
So you can let $y=x^3$ and you have: $y^2+5y+6=0$
Now you can solve by factorising.

Simultaneous equations

Simultaneous equations are equations where the solutions are pairs of values of x and y that satisfy both equations. You can solve them by elimination or by substitution.

Solving using elimination

Elimination means adding or subtracting the equations to eliminate one of the unknowns. These are the steps to follow:

- Make the coefficients of one of the unknowns the same. *(whichever seems easier)*
- Add or subtract the equations to eliminate one unknown (remember: **same** signs **subtract**, **different** signs **add**).
- Solve the new equation to find the first unknown.
- Substitute back into one of the original equations to find the other unknown.
- Check your values of x and y in both equations.

Example:

- **Solve simultaneously:** $\mathbf{3x - 2y = 4}$ (equation 1)
  $\mathbf{2x + 5y = 9}$ (equation 2)

Make the coefficient of x the same:	$\mathbf{6x - 4y = 8}$	(3)	*(equation 1)x2*
	$\mathbf{6x + 15y = 27}$	(4)	*(equation 2)x3*
<u>Subtract</u> (x's are the <u>same</u> sign):	$\mathbf{-19y = -19}$		(3) – (4)
Solve to find y:	$\underline{\mathbf{y = 1}}$		
Substitute into (2) to find x:	$\mathbf{2x + 5(1) = 9}$		*Replace y with 1*
	$\underline{\underline{\mathbf{x = 2}}}$		*Solve to find x*

Checks:

Equation (1):	$3x - 2y = 4$	$3(2) - 2(1) = 4$	$6 - 2 = 4$	✓
Equation (2):	$2x + 5y = 9$	$2(2) + 5(1) = 9$	$4 + 5 = 9$	✓

Solving using substitution

Substitution means substituting one of the equations into the other to get rid of one of the unknowns. These are the steps to follow (x and y have been used but it could be any letters):

- Rearrange one of the equations (if necessary) to make either **x or y the subject**.
- **Substitute** into the other equation.
- Solve the new equation to find x or y.
- Substitute back into your rearranged equation to find the value of the other letter.
- Check your values of x and y in both equations

Example:

- **Solve simultaneously:** $3y - 2x = 12$ (equation 1)
  $y + 5x = -13$ (equation 2)

Rearrange equation 2: $y = -5x - 13$ *(to make y the subject)*

Substitute y into equation 1: $3(-5x - 13) - 2x = 12$ *(Replace y with $-5x - 13$)*

Tidy up and solve for x:

$$\begin{aligned} -15x - 39 - 2x &= 12 \\ -17x &= 51 \\ \underline{\underline{x}} &\underline{\underline{= -3}} \end{aligned}$$

Substitute x to find y: *(Use rearranged version)*

$$\begin{aligned} y &= -5x - 13 \\ y &= -5(-3) - 13 \\ \underline{\underline{y}} &\underline{\underline{= 2}} \end{aligned}$$

Checks:

Equation (1):	$3y - 2x = 12$	$3(2) - 2(-3) = 12$	$6 + 6 = 12$	✓
Equation (2):	$y + 5x = -13$	$2 + 5(-3) = -13$	$2 - 15 = -13$	✓

One linear and one quadratic equation

When you have one linear and one quadratic equation the best way to solve them is by substitution.

e.g. Solve the simultaneous equations:

$$x + y = 8$$
$$x^2 + y^2 = 34$$

Rearrange the linear equation to make x or y the subject then substitute it into the quadratic equation. Try it! You should get the answers $x = 3$ and $y = 5$ OR $x = 5$ and $y = 3$

Linear inequalities

An inequality can use any of these signs: $<$ $>$ $\leq$ $\geq$ instead of the = sign. This means that your answer will be a range of values instead of just one value. You can solve them in the same way as you would if you had an = sign but there is one thing you need to remember:

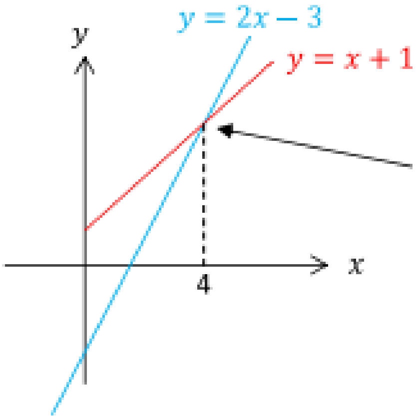
if you multiply or divide by a negative number you need to reverse the inequality sign.

e.g. Solve the inequality $2x - 3 > x + 1$ and sketch the outcome on a graph.

Calculation:

$$\begin{aligned} 2x - 3 &> x + 1 \\ 2x - x &> 1 + 3 \\ x &> 4 \end{aligned}$$

Graph:



This is the point where $2x - 3$ becomes greater than (above) $x + 1$

e.g. Solve the inequality: $\mathbf{5 - x \geq 2x - 1}$

$$\begin{aligned} -3x &\geq -6 \\ x &\leq 2 \end{aligned}$$

Notice that the inequality is reversed as you have divided by -3

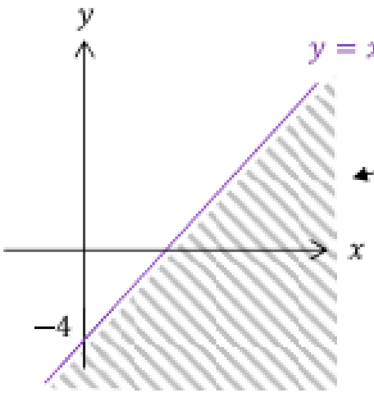
e.g. Solve the inequality: $\dfrac{x+3}{2} < \dfrac{x-2}{5}$

$$\begin{aligned} 5(x + 3) &< 2(x - 2) \\ 5x + 15 &< 2x - 4 \\ 3x &< -19 \\ x &< -\frac{19}{4} \end{aligned}$$

With all inequalities don't forget to check that your answer works! Try a value above and below your answer and check that the inequality is correct.

e.g. Sketch the inequality: $\mathbf{y < x - 4}$

For this type of question sketch the graph of $y = x - 4$ then show whether you want the area above or below the line.



In this case you want the area below the line because y is less than (<) the line.