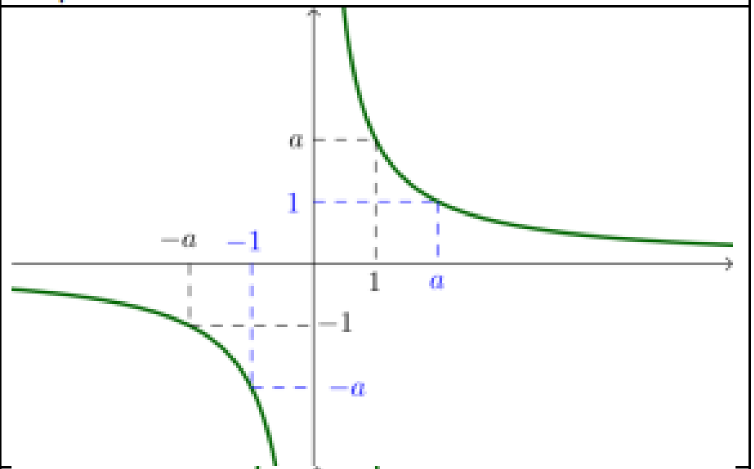
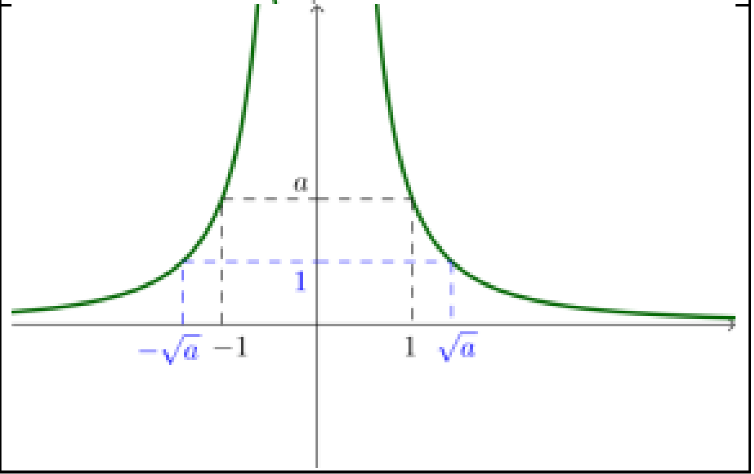
Y12 Pure Chapter 4— Graphs and Transformations

Graphs of functions

Functions or equations show the relationship between x and y and allow you to plot (or sketch) a graph. You might recognise some graphs straight away, and easily sketch them, but if not just find a few points (coordinates) and plot them to get an idea of what the graph looks like.

Think about the <u>shape</u> of the graph, where it crosses the <u>axes</u> and whether there are any <u>asymptotes</u>.

e.g. Think about the points shown on these 2 graphs and also where the graphs would be above and below each other if you plotted them on the same axes.

Equation	Think about	Graphs
$y = \frac{a}{x}$	**Asymptote** at $\boldsymbol{x = 0}$ (because you can't divide by 0) and at $\boldsymbol{y = 0}$ (because $a \div x \neq 0$). When x is positive, y is positive, and when x is negative, y is negative. You could think what this looks like by letting $a = 3$ (or any number) & plotting a few points.	
$y = \frac{a}{x^2}$	**Asymptote** at $\boldsymbol{x = 0}$ (because you can't divide by 0) and at $\boldsymbol{y = 0}$ (because $a \div x \neq 0$). When x is positive, y is positive, and when x is negative, y is positive. You could think what this looks like by letting $a = 3$ (or any number) & plotting a few points.	

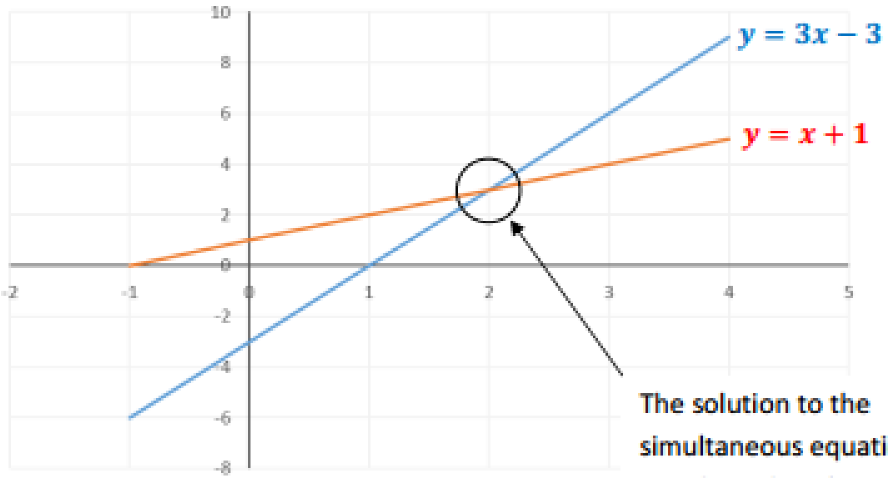
For help with sketching polynomials see 'Polynomials summary sheet'.

Interpret algebraic solution of equations graphically

When you have solved 2 (or more) equations simultaneously you can then plot the graphs, on the same axes, and show the intersection point (i.e. your solution) on the graphs.

Use intersection points of graphs to solve equations

You can plot 2 graphs on the same axes, and the intersection of the graphs will be the solution to the simultaneous equations.



The solution to the simultaneous equations $y = 3x - 3$ and $y = x + 1$

Understand and use proportional relationships and their graphs

Two quantities are proportional if they vary in the same way (e.g. if one doubles, the other doubles). You can draw a graph to show the relationship. The easy way to decide on the graph is to put a k in front of the x part of the equation.

e.g. $\boldsymbol{y \propto x}$ will become $\boldsymbol{y = kx}$ (a straight line graph with a gradient of k)
(y is proportional to x)

e.g. $\boldsymbol{y \propto x^2}$ will become $\boldsymbol{y = kx^2}$ (a curve (a larger value of k makes the curve steeper))
(y is proportional to x^2)

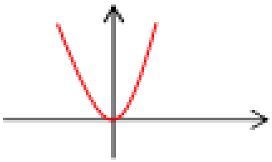
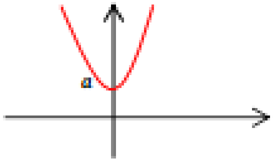
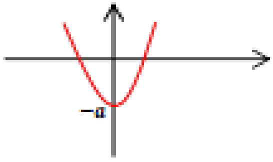
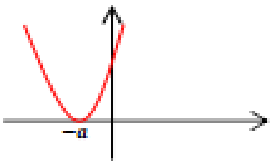
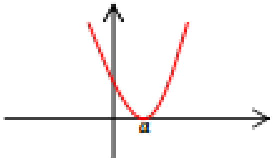
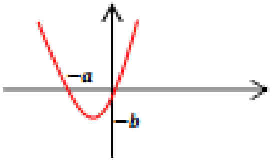
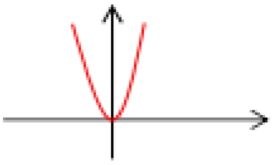
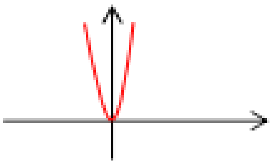
e.g. $\boldsymbol{y \propto \frac{1}{x}}$ will become $\boldsymbol{y = \frac{k}{x}}$ (a curve – shown on the 1st page)

This is called inverse proportion because as one value gets bigger the other gets smaller

Remember that for any of these graphs you can find k by substituting a point (x, y) and rearranging.

Transformations

You need to understand what happens to graphs when they are transformed. The following gives examples of some graph transformations that you need to learn:

e.g. graph: $y = x^2$	Original	
$y = x^2 + a$	Translation a units in y direction	
$y = x^2 - a$	Translation $-a$ units (i.e. down) in y direction	
$y = (x + a)^2$	Translation $-a$ (i.e. to the left) in x direction	
$y = (x - a)^2$	Translation a (i.e. to the right) in x direction	
$y = (x + a)^2 - b$	Translation $-a$ units in the x-direction and $-b$ units in the y-direction	
$y = ax^2$	Stretch scale factor a parallel to the y-axis	
$y = (ax)^2$	Stretch scale factor $\frac{1}{a}$ parallel to the x-axis	

Remember that these transformations work for all graphs, $y = x^2$ is just an example.
Try some yourself to see what happens. Replace a with a number and see how it affects your graph.

Y12 Pure Chapter 5— Straight Line Graphs

The equation of a straight line

There are various ways of writing the equation of a straight line. Remember that however you write it, it will contain an x and a y (or 2 different variables), because the equation is telling you the link between the variables.

$y = mx + c$	The most commonly used equation of a straight line, where m is the gradient and c is the y-intercept.
$y - y_1 = m(x - x_1)$	Useful if you are asked to find the equation of a straight line given the gradient and a point on the line. Substitute the gradient and the point and rearrange. N.B. if you find this equation difficult to remember you can always substitute the gradient and the point into the first equation and rearrange to find c.
$ax + by + c = 0$	Another way of writing a straight line. Useful when the gradient is a fraction. E.g. $y = \frac{2}{3}x + 4$ can be rearranged to $3y - 2x - 4 = 0$

Parallel or perpendicular

To decide whether 2 lines are parallel (like train tracks) or perpendicular (a right angle) you would look at the gradients.

Parallel	$m_1 = m_2$	The gradients are the same
Perpendicular (right angle)	$m_1 m_2 = -1$	Substitute the known gradient in and rearrange to find the perpendicular OR an easy way to find a perpendicular gradient is to think: *Turn it upside down and use the opposite sign*. e.g. gradient = 3, perpendicular gradient = $-\frac{1}{3}$

Finding the distance between 2 points

Use Pythagoras: Distance (length) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the coordinates of the midpoint of a line

Midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Form the equation of a straight line

Find the gradient (using $m = \frac{y_2 - y_1}{x_2 - x_1}$) then substitute the gradient and a given point into one of the forms shown above to find c.

Sketching a straight line

Sometimes you might be asked to sketch a straight line. If you are given 2 points on the line just plot them and join them together with a straight line.

$y = mx + c$	Plot the y-intercept then sketch the graph (remember to consider whether the gradient is positive or negative and how steep it should look. A gradient of 1 is at a $45°$ angle).
$ax + by + c = 0$	If you are given the equation in this form, it's probably easiest just to find the x and y intercepts. Remember the **x-intercept is when $y = 0$** and the **y-intercept is when $x = 0$**

Find the point of intersection of 2 lines

At a point of interception, the x and y coordinates are the same for both lines so you would solve them simultaneously (see 'Equations and Inequalities Summary Sheet' for how to do this).

The equation of a circle

The general equation of a circle is: $\boxed{(x-a)^2 + (y-b)^2 = r^2}$

Once you know this it is easy to find the centre and the radius.

$\boxed{\textbf{The centre is at } (a, b) \textbf{ and the radius = } r}$

e.g. Consider the circle $(x-3)^2 + (y+2)^2 = 9$

The centre is at $(3, -2)$ and the radius = 3

e.g. Consider the circle $x^2 + y^2 = 16$

The centre is at $(0, 0)$ and the radius = 4

This is a circle with the centre at the origin because $a = 0$ and b= 0

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Y12 Pure Chapter 6— Circles

Complete the square to find the centre and the radius

The equation of a circle could be given in a different form. To find the centre and the radius you would need to complete the square.

e.g. Find the centre and the radius of the following circle: $x^2 + y^2 + 10x - 4y = -4$

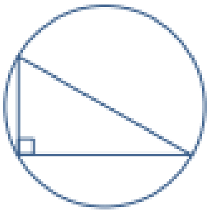
	$x^2 + y^2 + 10x - 4y = -4$
Collect x's and y's together:	$x^2 + \underset{\text{Half this}}{10x} + y^2 \underset{\text{Half this}}{- 4y} = -4$
Complete the square:	$(x + \boxed{5})^2 \underset{\text{Subtract back off}}{- 5^2} + (y - \boxed{2})^2 \underset{\text{Subtract back off}}{- 2^2} = -4$
Collect the numbers at the right hand side:	$(x+5)^2 + (y-2)^2 = -4 + 5^2 + 2^2$
Tidy up:	$(x+5)^2 + (y-2)^2 = 25$

In this form it is easy to see the centre and the radius of the circle. Centre = $(-5, 2)$ and radius = 5.

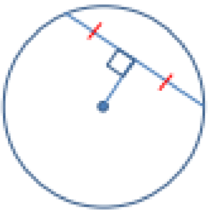
Circle properties

There are 3 properties of circles that you need to remember.

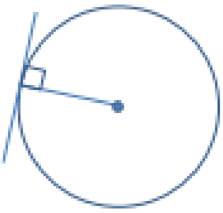
- The angle in a semi-circle is a right angle.



- If you draw a line from the centre of the circle, perpendicular to a chord, then the line will bisect the chord.



- Any tangent to a circle is perpendicular to the radius at the point where it touches the circle.



Find the point of intersection of a line and a circle

Solve simultaneously – remember that when a curve is involved (i.e. there are powers) it is easiest to use the method of substitution (see 'Equations and Inequalities Summary Sheet' for help).