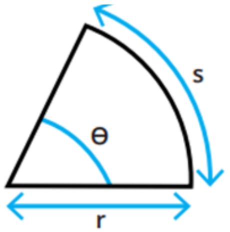
Topic 2: Motion in a Circle

Radians

The radian is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius. It is equivalent to about 57.3°.

This means that one whole rotation travels through 2π radians.

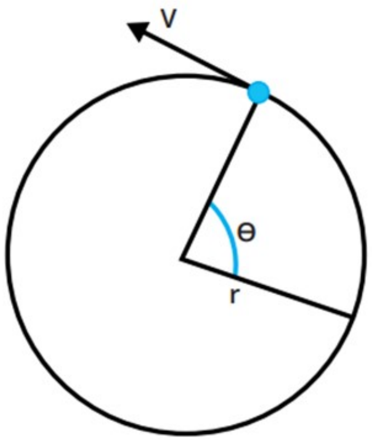


Angular Velocity

Angular velocity:

Motion in a circle or a cycle can be described by its time **period, T, the length of time for 1 cycle** and its **frequency, f, the number of cycles per second**.

$$T = \frac{1}{f}$$



For an object describing a circle at uniform speed, the **angular velocity**, ω, is equal to the angle θ swept out by the radius in time Δt divided by t.

$$\omega = \frac{\theta}{t}$$

As the time taken to complete a whole cycle, 2π, is T, the angular velocity can also be calculated by this equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

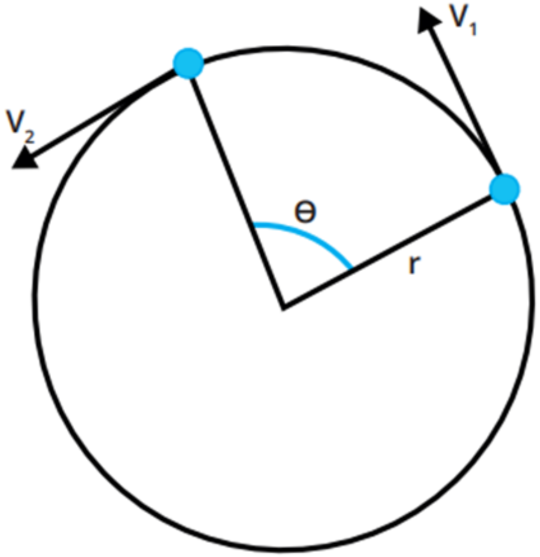
The relationship between the arc length and radius is arc length = $r\theta$ – this is the distance the object travels in time t. This means the speed of the object, v, can be calculated using this equation:

$$v = \omega r$$

Centripetal Acceleration and Force

This object is traveling with a constant speed, v, in a circular path. However, its **velocity** changes due to the **direction** changing. This means it must be accelerating due to a force acting on the object.

This is known as the **centripetal acceleration** as it is acting towards the centre of the circle.



It can be calculated in terms of v, and in terms of ω.

$$a = \frac{v^2}{r} = r\omega^2$$

From Newtons 2nd law; $F = ma$. Therefore, the force acting on the object can be calculated by these equations:

$$F = \frac{mv^2}{r} = mr\omega^2$$

v = velocity in m s^{-1}

F = force in N

T = period of one cycle in s

m = mass in kg

f = frequency in Hz

a = acceleration in m s^{-2}

ω = angular velocity in rad s^{-1}

r = radius in m