Y12 Pure Chapter 1— Algebraic Expressions

What do I need to be able to do?

By the end of this chapter you should be able to:

- Multiply and divide integer powers
- Expand a single term of brackets and collect like terms
- Expand the product of two or three expressions
- Factorise linear, quadratic and simple cubic expressions
- Know and use the laws of indices
- Simplify and use the rules of surds
- Rationalise denominators

Y12 – Chapter 1 Algebraic Expressions

Key words:

- Integer – A number with no fractional part (no decimals)
- Product – The answer when two or more values are multiplied together
- Surd – A number that can't be simplified to remove a square root (or cube root etc)
- Irrational – A real number that can NOT be made by dividing two integers eg π
- Rational – A number that can be made by dividing two integers
- Base – The number that gets multiplied when using an exponent (index/power)

Expanding and factorising

Expanding and factorising are the inverse of each other

Expanding brackets

$4x(2x+y) = 8x^2 + 4xy$

$(x+5)^3 = x^3 + 15x^2 + 75x + 125$

$(x+2y)(x-5y) = x^2 - 3xy - 10y^2$

Factorising

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Indices

An index (power) tells you how many times to multiply something by itself:

e.g. x^5 means $x \times x \times x \times x \times x$

There is a base and a power eg:

base → a^m ← power

Rule	Meaning
$a^m \times a^n = a^{m+n}$	To multiply 2 numbers with the same base you add the powers
$\frac{a^m}{a^n} = a^{m-n}$	To divide 2 numbers with the same base you subtract the powers
$(a^m)^n = a^{mn}$	To simplify a power inside and outside of a bracket you multiply the powers
$a^{-m} = \frac{1}{a^m}$	A negative power means find the reciprocal ("one over") so send everything to the bottom of a fraction
$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	A fractional power means a root. Denominator tells you the root and the numerator tells you the power.
$a^0 = 1$	Anything to the power of zero = 1
$a^1 = a$	Any number to the power of one stays the same

Surds

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified

e.g. $\sqrt{20}$ can be re-written as $\sqrt{4} \times \sqrt{5}$ which simplifies to $2\sqrt{5}$ (Perfect square)

Adding and subtracting surds

Remember to add or subtract like terms (ie. the rational numbers and the roots (of the same number))

e.g. $(7+3\sqrt{2})+(8-\sqrt{2}) = 15+2\sqrt{2}$ Add rational parts: $(7+8=15)$ Add roots: $(3\sqrt{2}-1\sqrt{2}=2\sqrt{2})$

Multiplying surds

If there is no rational part then multiplying is easy: e.g. $\sqrt{3}\times\sqrt{5} = \sqrt{15}$

If there is a rational part then multiply out the brackets

e.g.

$(5+\sqrt{3})\times(2-\sqrt{3}) = 10-5\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ tidies up to give $7-3\sqrt{3}$ (Remember that $\sqrt{3}\times\sqrt{3}=3$)

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction.

To rationalise the denominator just multiply the top and bottom of the fraction by the bottom of the fraction with the opposite sign in front of the root.

e.g. $\frac{3+\sqrt{5}}{2-\sqrt{5}}$

We are just finding an equivalent fraction by multiplying by 1 (just in disguise!)

$$\frac{3+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}} = \frac{11+5\sqrt{5}}{-1} = -11-5\sqrt{5}$$

Notice these are the same – but the sign in front of the root has changed

Changing the sign in front of the root makes the middle parts cancel each other out