Y13 Pure Chapter 1— Algebraic Methods

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use proof by contradiction to prove true statements
- Add, subtract, multiply and divide two or more algebraic fractions
- Convert an expression with linear factors in the denominator into partial fractions
- Convert an expression with repeated linear fractions in the denominator into partial fractions
- Divide algebraic expressions
- Convert an improper fraction into partial fraction form

Partial Fractions

Sometimes it can be useful to split a single algebraic fraction into two or more partial fractions.

Eg: $\frac{7x-13}{(x-3)(x+1)} = \frac{2}{x-3} + \frac{5}{x+1}$

When solving partial fractions, you start by setting your function equal to the unknown fractions you are trying to find. There are 3 different layouts which depend on the starting function

1) All linear terms in the denominator

$$\frac{7x-13}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

2) A repeated term in the denominator:

$$\frac{3x^2+7x-12}{(x-5)(x+2)^2} = \frac{A}{(x-5)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

3) *Improper fractions:

$$\frac{3x^2-3x-2}{(x-1)(x-2)} = A + \frac{B}{(x-1)} + \frac{C}{(x-2)}$$

Steps to solve:

1) Sett your functions equal to the correct unknown fraction as above
2) Add the fractions using a common denominator (this should be the same as the original denominator)
3) Set the numerators as equal
4) Substitute values for x that will, in turn, make each bracket zero and/or equate coefficients to create enough equations to find the values of A, B, C etc

*NB: You can either use algebraic division or the relationship F(x)= Q(x) x divisor + remainder to convert an improper fraction into a mixed fraction

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Key words:

- Contradiction – a disagreement between two statements which means that both cannot be true.
- Coefficient – A number used to multiply by a variable
- Improper algebraic fraction – One whose numerator has a degree equal to or larger than the denominator. It must be converted to a mixed fraction before you can express it in partial fractions

Pure Maths Year 2

Proof by Contradiction

To prove by contradiction you start by assuming that the statement is false. You then use logical steps until you contradict yourself by leading to something that is impossible. You can then conclude that your assumption was incorrect and that the original statement was true.

Eg: Prove by contradiction that $\sqrt{2}$ is irrational

Assumption: $\sqrt{2}$ is rational, therefore $\sqrt{2}$ can be written as $\frac{a}{b}$ where a and b are in their lowest form and that $\frac{a}{b}$ is in its lowest terms

$$\therefore 2 = \left(\frac{a}{b}\right)^2$$
$$2 = \frac{a^2}{b^2}$$
$$\therefore 2b^2 = a^2$$

This means that a^2 is even which means that a is even.
If a is even then it can be expressed as 2k

$$\therefore a^2 = 2b^2$$
$$(2k)^2 = 2b^2$$
$$4k^2 = 2b^2$$
$$2k^2 = b^2$$

This means that b^2 is even which means that b is even.

Conclusion: If a and b are both even then the have a common factor of 2 so $\frac{a}{b}$ cannot be a fraction in its lowest terms which is a contradiction. This means that the original assumption is not correct and therefore $\sqrt{2}$ is irrational