Y13 Pure Chapter 12—Vectors

What do I need to be able to do?

By the end of this chapter you should be able to:

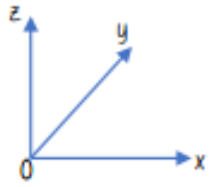
- Understand 3D Cartesian coordinates
- Use vectors in three dimensions
- Use vectors to solve geometric problems
- Model 3D motion in mechanics with vectors

Y13 – Chapter 12 Vectors

Key words:

- Coplanar vectors – Vectors in the same plane
- Magnitude– The size of the vector

3D Coordinates



When visualising 3D coordinates, think of the x and y axis drawn on a flat surface with the z axis sticking up from the flat surface.

The distance from the origin to the point (x, y, z) is:

$$\sqrt{x^2 + y^2 + z^2}$$

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

3D Vectors

Unit vectors along the x, y and z axes are denoted by i, j and k respectively

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

For any 3D vector $\boldsymbol{pi + qj + rk} = \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{q} \\ \boldsymbol{r} \end{pmatrix}$

Parallel Vectors in 3D

If a, b, and c are 3D, non-coplanar vectors (not in the same plane) then you can compare coefficients on both sides of an equation:

Eg

If

$$\boldsymbol{pi + qj + rk = ui + vj + wk}$$

Then: p=u, q=v and r=w

Magnitude and Direction

Vector a = $\boldsymbol{pi + qj + rk}$

Magnitude of vector a:

$$|\boldsymbol{a}| = \sqrt{p^2 + q^2 + r^2}$$

Direction of vector a:

The angle with the x-axis: $\cos\theta_x = \frac{\boldsymbol{p}}{|\boldsymbol{a}|}$

The angle with the y-axis: $\cos\theta_y = \frac{\boldsymbol{q}}{|\boldsymbol{a}|}$

The angle with the z-axis: $\cos\theta_z = \frac{\boldsymbol{r}}{|\boldsymbol{a}|}$

Pure Maths Year 2