Y13 Pure Chapter 3—Sequences and Series

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the nth term of an arithmetic sequence and a geometric sequence
- Prove and use the formula for the sum of the first n terms of an arithmetic series
- Prove and use the formula for the sum of a finite geometric series
- Prove and use the formula for the sum to infinity of a convergent geometric series
- Use sigma notation
- Generate sequences from recurrence relations
- Model real life situations

Y13 – Chapter 3 Sequences and Series

Key words:

- Sequence – A list of numbers or objects in a special order
- Series – The sum of terms in a sequence
- Arithmetic sequence – A sequence made by adding the same value each time
- Geometric sequence – A sequence made by multiplying by the same value each time. There is a common ratio between consecutive terms
- Arithmetic series– the sum of the terms of an arithmetic sequence
- Geometric series – the sum of the terms in a geometric sequence
- Common ratio – The amount we multiply by each time in a geometric sequence
- Converging sequence/series – A sequence/series converges when it keeps getting closer and closer to a certain value
- Divergent series – does not settle towards a certain value. When a series diverges it goes off to infinity, minus infinity, or up and down without settling towards some value.

Pure Maths Year 2

Arithmetic Sequences and Series

The formula for the nth term of an arithmetic sequence is:

$$u_n = a + (n-1)d$$

u_n is the nth term
a is the first term
d is the common difference

The formula for the sum of the first n terms of an arithmetic series is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

It can also be written as:

$$S_n = \frac{n}{2}(a + l)$$

a is the first term
d is the common difference
l is the last term

Sigma Notation

Σ means "the sum of". You write on the top and bottom to show which terms you are summing.

Eg:

$$\sum_{r=1}^{5}(2r-3) = -1 + 1 + 3 + 5 + 7$$

This tells you that you are summing the expression in brackets with $r=1$ up to $r=5$

Substitute $r=1$, $r=2$, $r=3$, $r=4$ and $r=5$ into the expression in brackets to find the 5 terms in this arithmetic series

Recurrence Relations

The next term in the sequence is the function of the previous term

$$u_{n+1} = f(u_n)$$

Geometric Sequences and Series

The formula for the nth term of a geometric sequence is:

$$u_n = ar^{n-1}$$

u_n is the nth term
a is the first term
r is the common ratio

The formula for the sum of the first n terms of a geometric series is:

$$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

It can also be written as:

$$S_n = \frac{a(r^n-1)}{r-1}, r \neq 1$$

a is the first term
r is the common ratio

Sum to Infinity

As n tends to infinity, the sum of a geometric series is called the sum to infinity.
A geometric series is convergent only when $|r| < 1$, where r is the common ratio

The formula for the sum to infinity of a convergent series is:

$$S_\infty = \frac{a}{1-r}$$

a is the first term
r is the common ratio