Y13 Pure Chapter 4—Binomial Expansion

What do I need to be able to do?

By the end of this chapter you should be able to:

- Expand $(1+x)^n$ for any rational constant n and determine the range of values of x for which the expansion is valid
- Expand $(a+bx)^n$ for any rational constant n and determine the range of values of x for which the expansion is valid
- Use partial fractions to expand fractional expressions

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Key words:

- Infinite series – The sum of infinite terms that follow a rule

Pure Maths Year 2

The Binomial Expansion

$(a+b)^n$ when n is a fraction or a negative number (ie. NOT a positive integer) there will be an infinite number of terms. This means that the binomial expansion can only be used when $-1 < x < 1$

When n is a fraction or a negative number the following form of the binomial expansion should be used:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{r}x^r + \cdots, (|x| < 1, n \in \mathbb{R})$$

The expansion is valid when $|x| < 1$

The expansion of $(1+bx)^n$ is valid for $|bx| < 1$ or $|x| < \frac{1}{b}$

We can use the expansion of $(1+x)^n$ to expand $(a+bx)^n$ by taking out a factor of a^n out of the expression

$$(a+bx)^n = (a\left(1+\frac{b}{a}x\right))^n = a^n(1+\frac{b}{a}x)^n$$

The expansion of $(a+bx)^n$ is valid for $\left|\frac{b}{a}x\right| < 1$ or $|x| < \frac{a}{b}$

Partial Fractions

We can use partial fractions to simplify the expansions of more difficult expressions

E.g.

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions

b) Hence show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2 - \frac{7x}{2} + \frac{11}{4}x^2 - \frac{25}{8}x^3$

c) State the range of values of x for which the expansion is valid

a) $\frac{4-5x}{(1+x)(2-x)} \equiv \frac{A}{1+x} + \frac{B}{2-x}$

$\equiv \frac{A(2-x)+B(1+x)}{(1+x)(2-x)}$

$4 - 5x \equiv A(2-x) + B(1+x)$

Substitute $x = 2$:

$4 - 10 = A \times 0 + B \times 3$

$B = -2$

Substitute $x = -1$:

$4 + 5 = A \times 3 + B \times 0$

$A = 3$

b) $\frac{4-5x}{(1+x)(2-x)} = \frac{3}{1+x} - \frac{2}{2-x}$

$= 3(1+x)^{-1} - 2(2-x)^{-1}$

The expansion of $3(1+x)^{-1} = 3 - 3x + 3x^2 - 3x^3 + \cdots$

The expansion of $2(2-x)^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots$

Hence $\frac{4-5x}{(1+x)(2-x)} = (3 - 3x + 3x^2 - 3x^3) - \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}\right)$

$= 2 - \frac{7}{2}x + \frac{11}{4}x^2 - \frac{25}{8}x^3$

c) $\frac{3}{1+x}$ is valid if $|x| < 1$ $\quad$ $\frac{2}{2-x}$ is valid if $|x| < 2$

So the expansion is valid when $|x| < 1$