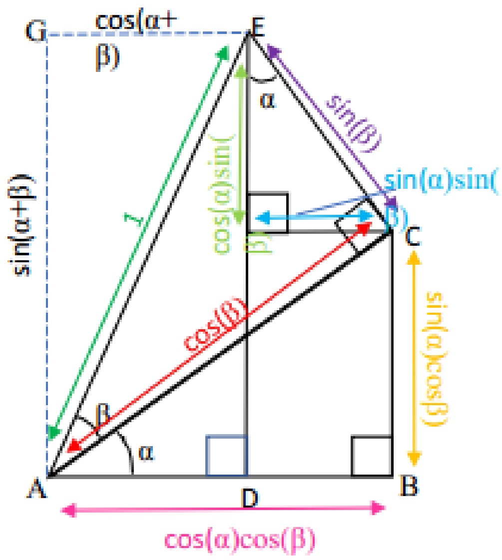
Y13 Pure Chapter 7—Trigonometry and Modelling

What do I need to be able to do?

By the end of this chapter you should be able to:

- Prove and use the addition formulae
- Understand and use the double angle formulae
- Solve trigonometric equations using addition and double angle formulae
- Write expressions in the form $R\cos(\theta \pm \alpha)$ and $R\sin(\theta \pm \alpha)$
- Prove trigonometric identities
- Model real life situations

Proof of the Addition Formulae



Using the properties of sine and cosine we can label the diagram as above.

Using triangle ADE:
DE = sin(α+β)
AD = cos (α+β)

DE = DF + FE
∴ sin(α+β) = sin(α)cos(β) + cos(α)sin(β)

AD = AB – DB
∴ cos(α+β) = cos(α)cos(β) – sin(α)sin(β)

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Addition Formulae

Sometimes are known as the compound angle formulae

$$sin(A+B) \equiv sinAcosB + cosAsinB$$
$$sin(A-B) \equiv sinAcosB - cosAsinB$$

$$cos(A+B) \equiv cosAcosB - sinAsinB$$
$$cos(A-B) \equiv cosAcosB + sinAsinB$$

$$tan(A+B) \equiv \frac{tanA + tanB}{1 - tanAtanB}$$

$$tan(A-B) \equiv \frac{tanA - tanB}{1 + tanAtanB}$$

Simplifying acos(x) ± bcos(x)

Sometimes known as the harmonic form. You can write expressions in the form acos(x) ± bcos(x) as a function of sine or cosine only.

$acos\theta \pm bsin\theta$ can be written as either:

$Rsin(x \pm \alpha)$ where R>0 and 0< α<90

$Rcos(x \pm \beta)$ where R>0 and 0< β<90

Where $Rcosa = a$ and $Rsina = b$ and $R = \sqrt{a^2 + b^2}$

Double Angle Formulae

You can use the addition formulae to derive the following double angle formulae:

$$sin(2A) \equiv 2sinAcosA$$

$$cos(2A) \equiv cos^2A - sin^2A \equiv 2cos^2A - 1 \equiv 1 - 2sin^2A$$

$$tan(2A) \equiv \frac{2tanA}{1 - tan^2A}$$

Pure Maths Year 2