Y13 Pure Chapter 8—Parametric Equations

What do I need to be able to do?

By the end of this chapter you should be able to:

- Convert parametric equations into Cartesian form
- Understand and use parametric equations of curves and sketch parametric curves
- Solve problems involving parametric equations
- Use parametric equations in modelling

Y13 – Chapter 8 Parametric Equations

Pure Maths Year 2

Key words:

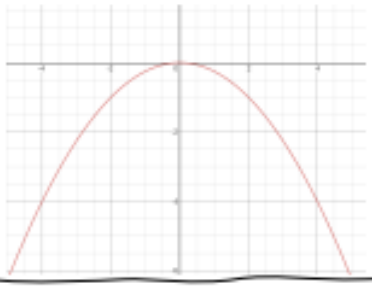
- Cartesian equations – Gives a direct relationship between x and y
- Parametric equations – Uses a third variable (usually t or θ) to define x and y

Sketching parametric equations of curves

When sketching a parametric equation, sub in values of t to find x and y values and then sketch as normal

Sketch the curve defined by $x=2t$ and $y=-t^2$ between $t=-3$ and 3.

T	-3	-2	-1	0	1	2	3
X	-6	-4	-2	0	2	4	6
Y	-9	-4	-1	0	-1	-4	-9



Converting between parametric and cartesian equations

Combine the two equations by rearranging one of them to make t the subject and then substitute into the other equation

OR

Rearrange both equations to make t the subject and then equate the two equations

Eg: Convert the following parametric equations into cartesian form

$x = t + 3$ $y = 2t^2$

$x = t + 3 \rightarrow t = x - 3$

$y = 2(x - 3)^2$

OR

$x = t + 3$ $y = 2t^2$

$x = t + 3 \rightarrow t = x - 3$

$y = 2t^2 \rightarrow t = \sqrt{(y/2)}$

$\sqrt{(y/2)} = x - 3$

$y/2 = (x - 3)^2$

$y = 2(x - 3)^2$

If your parametric equations contains trigonometric functions, first find an identity that connects then rearrange the parametric equations so that you can substitute into the identity

Calculus with parametric equations*

* This section actually appears in your text book in chapters 9 and 11

Differentiation:

If $x = f(t)$ and $y = g(t)$

Then:

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

Integration:

If $x = f(t)$ and $y = g(t)$

Then:

$$\int y\,dx = \int y \frac{dx}{dt} dt$$

Remember to adjust limits if you are using definite integration

Domain and range

For parametric equations $x = p(t)$ and $y = q(t)$ with Cartesian equation $y = f(x)$

- The domain of $f(x)$ is the range of $p(t)$
- The range of $f(x)$ is the range of $q(t)$