Y13 Pure Chapter 9—Differentiation

What do I need to be able to do?

By the end of this chapter you should be able to:

- Differentiate trigonometric functions
- Differentiate exponentials and logarithms
- Use the chain, product and quotient rules
- Use the second derivative to describe a function's behaviour
- Solve problems involving connected rates of change
- Construct differential equations
- Differentiate parametric equations (see parametric equations sheet)

Differentiating Trigonometric functions

If $y = \sin kx$, then $\frac{dy}{dx} = k\cos kx$

If $y = \cos kx$, then $\frac{dy}{dx} = -k\sin kx$

If $y = \tan kx$, then $\frac{dy}{dx} = k\sec^2 kx$

If $y = \operatorname{cosec} kx$, then $\frac{dy}{dx} = -k\operatorname{cosec} kx\cot kx$

If $y = \sec kx$, then $\frac{dy}{dx} = -k\sec kx\tan kx$

If $y = \cot kx$, then $\frac{dy}{dx} = -k\operatorname{cosec}^2 kx$

If $y = \arcsin x$, then $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

If $y = \arccos x$, then $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

If $y = \arctan x$, then $\frac{dy}{dx} = \frac{1}{1+x^2}$

Implicit Differentiation

Use when equations are difficult to rearrange into the form $y = f(x)$

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$$

$$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$$

Second Derivatives

The function f(x) is concave on a given interval if and only if f''(x) ≤ 0 for every value of x in the value in that interval

The function f(x) is convex on a given interval if and only if f''(x) ≥ 0 for every value of x in that interval

A point of inflection is a point at which f''(x) changes sign

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Key words:

- Concave – Curves inwards
- Convex – Curves outwards

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Chain, Product and Quotient Rules

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Where y is a function of u and u is another function of x

In function notation:

If $y = (f(x))^n$ then $\frac{dy}{dx} = n(f(x))^{n-1}f'(x)$

If $y = f(g(x))$ then $\frac{dy}{dx} = f'(g(x))g'(x)$

Product Rule:

If y = uv where u and v are functions of x, then:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

In function notation:

If $f(x) = g(x)h(x)$ then:

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

Quotient Rule:

If y = u/v where u and v are functions of x, then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

In function notation:

If $f(x) = g(x)/h(x)$ then:

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2}$$

Differentiating Exponential and Logarithms

If $y = e^{kx}$, then $\frac{dy}{dx} = ke^{kx}$

If $y = \ln x$, then $\frac{dy}{dx} = \frac{1}{x}$

If $y = a^{kx}$, where k is a real constant and a>0, then $\frac{dy}{dx} = a^{kx}k\ln a$