Module 5 Section 2: Circular Motion and Oscillations

Radians

Objects in circular motion travel through angles — these angles are usually measured in radians.

The angle in **radians**, θ, is equal to the arc-length divided by the radius of the circle (see Figure 1).

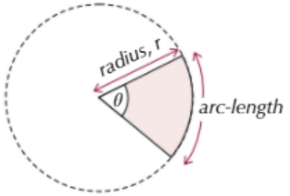


Figure 1: The radius of a circle, and the angle and arc-length of a sector.

In other words:

arc length $= r\theta$

$$\text{angle in radians} = \frac{\pi}{180°} \times \text{angle in degrees}$$

Angular Velocity

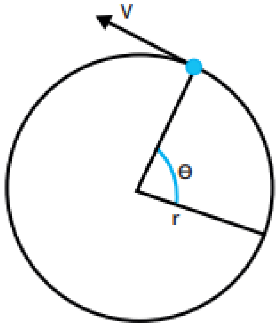
Angular velocity:

Motion in a circle or a cycle can be described by its time **period, T, the length of time for 1 cycle** and its **frequency, f, the number of cycles per second**.

$$T = \frac{1}{f}$$

For an object describing a circle at uniform speed, the **angular velocity**, ω, is equal to the angle θ swept out by the radius in time Δt divided by t.

$$\omega = \frac{\theta}{t}$$



As the time taken to complete a whole cycle, 2π, is T, the angular velocity can also be calculated by this equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

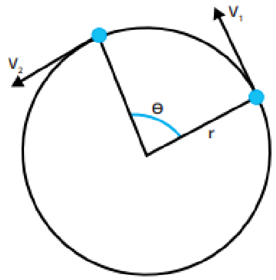
The relationship between the arc length and radius is arc length = $r\theta$ – this is the distance the object travels in time t. This means the speed of the object, v, can be calculated using this equation:

$$v = \omega r$$

Centripetal Force and Acceleration

This object is traveling with a constant speed, v, in a circular path. However, its **velocity** changes due to the **direction** changing. This means it must be accelerating due to a force acting on the object.

This is known as the **centripetal acceleration** as it is acting towards the centre of the circle.



It can be calculated in terms of v, and in terms of ω.

$$a = \frac{v^2}{r} = r\omega^2$$

From Newtons 2nd law; $F = ma$. Therefore, the force acting on the object can be calculated by these equations:

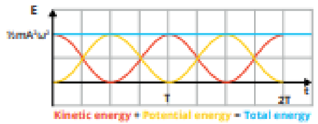
$$F = \frac{mv^2}{r} = mr\omega^2$$

Energy

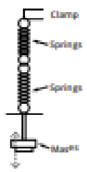
A body during SHM will have a constant energy, but it will transfer from potential energy to kinetic energy.

The kinetic energy of a body during SHM can be calculated using $E_k = \frac{1}{2}mv^2$ and therefore $E_{k\,max} = \frac{1}{2}mvA^2\omega^2$.

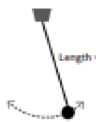
When the kinetic energy is at its maximum, the potential energy will be zero. Therefore, the energy of the body in SHM is always equal to $\frac{1}{2}mvA^2\omega^2$.



Two common examples of SHM are **masses on a spring** and a **simple pendulum**. The equations to calculate the period is given for each.



$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{l}{g}}$$

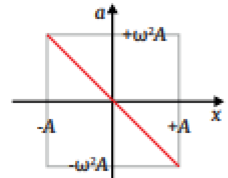
You must be able to describe using the apparatus to calculate k or g, as well as using the equations.

Simple Harmonic Motion

Simple harmonic motion (SHM) occurs when an object moves such that **its acceleration is always directed toward a fixed point and is proportional to its distance from the fixed point**. This can be expressed in this equation:

$$a = -\omega^2 x$$

where $-\omega^2$ is a constant. Plotting a graph of a against x would give this shape, this is the same shape for all SHM.



A, is amplitude, the **maximum value of the displacement**.

Other key definitions are; the period, T, which is the **time taken for one complete cycle**, and frequency, f, which is the **number of oscillations per second**. (See component 3.1)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Another way of defining SHM is where the motion of a point whose displacement, x, changes with time, t, according to $x = A\cos(\omega t + \varepsilon)$, where A, ω and ε are constants.

ε is the phase constant. It is normally 0 or $\pi/2$ depending on the displacement at time, $t = 0$.

If at $t = 0$ the displacement is at its maximum, then ε = 0 and if at t = 0, the displacement is 0, then $\varepsilon = \pi/2$.

The velocity during the motion can be calculated using the following equation:

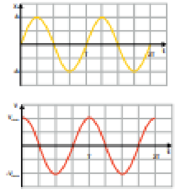
$$v = -A\omega\sin(\omega t + \varepsilon)$$

The maximum value for $\sin(\omega t+\varepsilon) = 1$. Therefore, the maximum velocity, $v_{max} = A\omega$.

Graphical Representations

The equations show that both x and v vary sinusoidally with time during SHM.

Note that v is at a maximum when $x = 0$.



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Calculations

So for an object undergoing SHM, the displacement can be described by:

$$x = A\cos(\omega t) \quad \text{or} \quad x = A\sin(\omega t)$$

x = displacement in m
A = amplitude in m
t = time in s
ω = angular frequency in rads^{-1}

So the acceleration of the object is:

$$a = -\omega^2 x$$

a = acceleration in ms^{-2}
x = displacement in m
ω = angular frequency in rads^{-1}

Since $a = -\omega^2 A\cos(\omega t)$ or $a = -\omega^2 A\sin(\omega t)$, the object's acceleration has a maximum magnitude of $a_{max} = \omega^2 A$. This occurs when the object's displacement is at its maximum magnitude — i.e. when $x = \pm A$:

$$a_{max} = \omega^2 A$$

a_{max} = magnitude of the max acceleration in ms^{-2}
A = amplitude in m
ω = angular frequency in rads^{-1}

You also need to know the equation for the velocity of an object moving with SHM. Don't worry about the derivation of v, it's pretty complicated. You just need to know that it is given by:

$$v = \pm\omega\sqrt{A^2 - x^2}$$

v = velocity in ms^{-1}
x = displacement in m
ω = angular frequency in rads^{-1}
A = amplitude in m

$$\text{Max speed} = v_{max} = \omega A$$

v_{max} = max speed in ms^{-1}
A = amplitude in m
ω = angular frequency in rads^{-1}

Investigating SHM

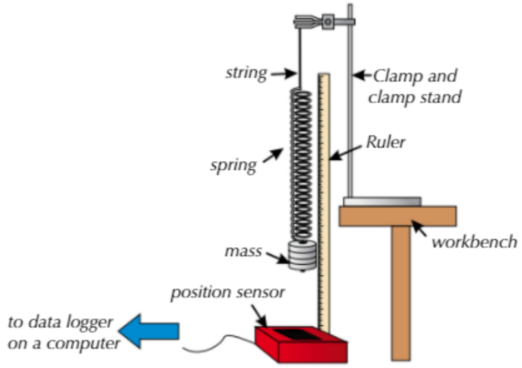


***Figure 1:** Experimental set-up for investigating the motion of a mass oscillating on a spring.*

Lift the mass slightly and release it — this will cause the mass-spring system to start oscillating with simple harmonic motion.

To make sure your experiment is repeatable, place a ruler behind the spring to measure how far you raise the mass. Make sure your eye is level with the mass when you take the measurement.

As the mass oscillates, the position sensor will measure the displacement of the mass over time. The computer can be set to record this data automatically.

Let the experiment run until you've got a good amount of data (at least ten complete oscillations).

Free Oscillations

Free oscillations occur when an oscillatory system (such as a mass on a spring, or a pendulum) is displaced and released. The frequency of the free oscillations is called the system's **natural frequency**.

In reality, the oscillations will not stay at the same amplitude indefinitely but, will decrease over time due to resistive forces. This is known as **damping**.

Damping is very important in a number of situations. For example, in a car's shock absorbers, the vibration caused by going over a bump does not last long (otherwise all the passengers will feel unwell). A specific case is **critical damping**, where the resistive forces on the system are just large enough to **prevent oscillations** occurring at all when the system is displaced and released.

Light and heavy damping

Lightly damped systems take a long time to stop oscillating, and their amplitude only reduces a small amount each period. Heavily damped systems take less time to stop oscillating, and their amplitude gets much smaller each period.

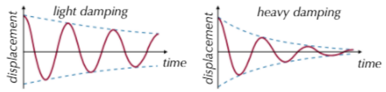
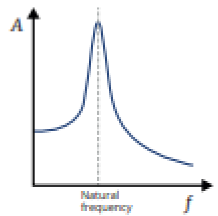


***Figure 5:** Graphs showing the effect of light and heavy damping.*

Resonance

When a sinusoidally varying driving force is applied to an oscillating system, if the **frequency of the applied force is equal to the natural frequency** of the system, the amplitude of the resulting oscillations is large. This is **resonance**.



Resonance can be **useful**, for example, microwaves with a frequency similar to the natural frequency of water molecule vibration are used in microwave ovens to heat the water molecules.

However, it is **not always useful**. For example, the frequency of people walking on the millennium bridge was close to the natural frequency of the bridge and caused it to oscillate dangerously.