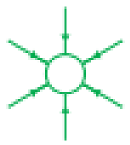
Module 5 Section 3: Gravitational Fields

Gravitational field

A gravitational field is a force field generated by any object with mass which causes any other object with mass to experience an attractive force.

Field lines:



Note the field outside a spherical body is the same shape as if all the mass was concentrated at the centre.

Is always attractive.

Newton's Laws of Gravitation

Newton's law of gravitation says that the force acting between two point masses (or spherical masses) is proportional to the product of their masses and inversely proportional to the square of the distance between their centres of mass:

$$F = -\frac{GMm}{r^2}$$

- G = gravitational constant = 6.67×10^{-11} Nm²kg⁻²
- M = mass causing the force in kg
- F = force acting on mass m due to mass M in N
- m = mass the force is acting on in kg
- r = distance between the centres of the two masses in m

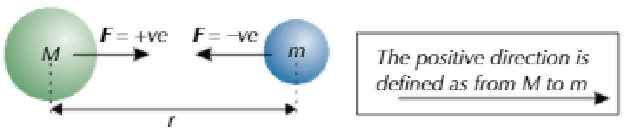


Figure 4: The forces acting on the two masses are equal but opposite.

We sometimes only consider the force acting on the smaller object because that's the one that experiences a greater acceleration — $a = \frac{F}{m}$, so as m becomes bigger, a becomes smaller.

The law of gravitation is an **inverse square law**: $F \propto \frac{1}{r^2}$

Gravitational Field Strength

Gravitational field strength, g, is the force per unit mass. Its value depends on where you are in the field. There's a really simple equation for working it out:

$$g = \frac{F}{m}$$

- g = gravitational field strength in Nkg⁻¹
- F = force experienced by a mass, m, in the gravitational field in N
- m = mass in kg

The value of **g** at the Earth's surface is approximately 9.81 Nkg⁻¹

Radial fields

Point masses (or spherical masses) have radial gravitational fields (see page 297). The value of **g** depends on the distance r from the centre of the mass M.

$$g = -\frac{GM}{r^2}$$

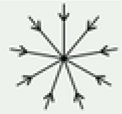
- G = gravitational constant in Nm²kg⁻²
- M = mass of object causing the gravitational field in kg
- g = gravitational field strength in Nkg⁻¹
- r = distance from the centre of mass M in m

Key points

Gravitational field strength, g, is force per unit mass.

Newton's law of gravitation for the force between two point masses is an inverse square law. $F \propto \frac{1}{r^2}$

The gravitational field lines for a point mass...



Gravitational potential, V_g, is potential energy per unit mass and is zero at infinity.

Gravitational field strength: $g = \frac{F}{m}$

Newton's law of gravitation: $F = -\frac{GMm}{r^2}$

Gravitational field strength for a radial field: $g = -\frac{GM}{r^2}$

Gravitational potential: $V_g = -\frac{GM}{r}$

Gravitational Potential and Energy

Gravitational potential is defined as the **work** done **per unit mass** in bringing a mass from infinity to that point.

$$V_g = -G\frac{M}{r}$$

Therefore, electrical potential energy is given by:

$$PE = -G\frac{M_1 M_2}{r}$$

If the field is uniform, i.e. **g is constant**, then the gravitational potential energy can also be calculated using:

$$\Delta U_p = mg\Delta h$$

Escape Velocity

The **escape velocity** is defined as the velocity needed so an object has just enough kinetic energy to escape a gravitational field. In other words, it's the minimum speed an unpowered object needs in order to leave the gravitational field of a planet and not fall back towards the planet due to gravitational attraction.

So kinetic energy lost = gravitational potential energy gained, and so:

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

Cancelling out m and rearranging for v gives the escape velocity:

$$v = \sqrt{\frac{2GM}{r}}$$

- v = escape velocity in ms⁻¹
- G = gravitational constant = 6.67×10^{-11} Nm²kg⁻²
- r = distance of the object from the centre of mass of the planet in m
- M = mass of the planet in kg